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Editorial

Introducing the volume

1. How this tribute to K. Itô was set up

Soon after the sad news about Professor Kiyosi Itô's passing away, on November 10th, 2008, the Editorial Board of the journal SPA decided to publish a special issue dedicated to K. Itô's memory and achievements. With the invitation to be guest editor, one of us (M.Y.) felt at the same time honored, but also experienced some awe, when facing the idea of having to organize a volume which ought to reflect the immensity of K. Itô's achievements in Probability Theory.

However, with time, the following more positive thoughts emerged:

(a) K. Itô was one of the giants in Probability Theory in the 20th century. Hence, several major probability journals would plan to honor K. Itô's memory by devoting a special issue about his work; thus the special SPA issue did not need to cover the entire spectrum of K. Itô's discoveries;

(b) Inspired by K. Itô's main attitude: "go to the essentials", the option was made for the presentation, illustration and applications of the two Itô masterpieces, namely: *Stochastic Calculus* and *Excursion Theory*;

(c) The choice of the authors seemed to come to our minds naturally, involving four direct former students of K. Itô: Professors Masatoshi Fukushima, Nobuyuki Ikeda (together with Setsuo Taniguchi), Hiroshi Kunita and Shinzo Watanabe, and four French probabilists: Professors Jean Bertoin, Philippe Biane, Jean-François Le Gall and Wendelin Werner. The former give, in particular, a historical perspective to the development of Probability Theory in Japan, under the guidance of K. Itô, whereas the latter show, in different ways, how their research field benefited from Itô's superb tools, as well as how they were able to imagine and exploit their own tools, often inspired by K. Itô's. The detailed contents of the eight papers are described at the end of the introduction to this special issue;

(d) It is better not to be alone in this kind of project; so, it was thought best that M.Y. and M.E.V. coedit this Tribute.

Each author has done an excellent job, and hoping our "hypothesis" in (a) above is correct, other probability journals will also develop many facets of K. Itô's work, which are missing here, as well as other applications of e.g. his wonderful stochastic calculus.

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Itô's heritage will of course remain with us: Itô has built the Giants' Causeway[1] between, on one hand Kolmogorov and Lévy, and on the other hand, the probability theories of the 21st century, which we begin to see unfolding: random times, planar maps, combinatorics, SLE, random matrices, black noise and last but not least, mathematical finance, although there, W.B. Yeats' appreciation may be the right one: "A terrible beauty is born".[2]

Each paper underwent several iterations, which were carefully read by the guest editors.

Many thanks to all authors for their dedication and efficiency. We are grateful to them for these special contributions. D. Stroock inspired our completion of this small piece, but is not to be blamed for the Irish imports. . . .

A view of the Giants' Causeway (: The Honeycomb and Wishing chair).

2. What you will not find in this volume

It is rather easy to answer the informal question found in the above title: you will not find a global analysis of K. Itô's achievements.

However, such analysis, up to 1987, is already presented in a masterful way by D. Stroock and S. Varadhan in their Introduction to Itô's Selected Papers (Springer, 1987), followed by an impressive, and very informative, foreword by Itô himself.

For papers published by Itô between 1987 and 1996, which are devoted mainly to Itô's explanation of Malliavin Calculus, a very nice exposition is found on page x of the book: *Itô's Stochastic Calculus and Probability Theory*, edited by N. Ikeda, S. Watanabe, M. Fukushima, H. Kunita (Springer, 1996).

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[1] Until the 19th century, it was thought that these basalt rocks off the Northern Ireland coast had been used by giants as stepping stones between Scotland and Ireland.

[2] This is the last line of W.B. Yeats (1865–1939) famous poem *Easter 1916*.

Despite the existence of these thorough overviews of Itô's achievements, it may be worth selecting a few topics (other than Stochastic Integration and Excursion Theory) which have been studied in depth by K. Itô:

- *stationary processes* (4, 6, 19, 29: the famous paper with M. Nisio);
- *kinematic theory of turbulence* [5][3,4];
- *diffusion on manifolds* (12, 13, 18, 26, 42: Stochastic parallel displacement), all of which may be considered as seeds for the many studies of stochastic differential geometry which blossomed both in Great Britain and in France at the end of the seventies and beginning of the eighties;
- *a refinement of Rice's formula* [28];
- *relations between Wiener measure and Feynman integral* [23, 32];
- the first paper, to our knowledge, asking about *enlargement* of filtrations (47, already in 1976!) where Itô wonders about the meaning of $\int_0^t B_1 \mathrm{d}B_s$;
- *infinite dimensional diffusions and SDEs* [50, 54].

Each of these works has been at the origin of a vast literature, and, no doubt, would still be of great interest to be scrutinized, both from the historical, and today's state of the art, perspectives.

But, as seemed most reasonable to us, this may be left, among other aspects, to other probability journals eager to honor the many achievements of K. Itô, some of which have just been evoked here.

Another view of the Giants' Causeway (: Giants' Organ, Spanish Head).

[3] About this paper, we received the following comments from Professor M. Lesieur, a specialist of turbulence, and a member of the Académie des Sciences: How could Itô work in this war period? It is not quite right that the turbulence is Gaussian, as assumed by K. Itô. However, there is the quasi-normal approximation of Millionschtchikov (1941), a student of Kolmogorov, for the 4th moments of speeds.

[4] The numbers used in these two pages refer to those of the just preceding Bibliography of K. Itô.

3. Eight mathematical papers

Here is first a summary of the contents of each of the eight papers, largely inspired from the abstracts of these papers:

M. Fukushima identifies the Dirichlet form of X, a generic one-dimensional diffusion process, on $L^2(m)$, where m denotes the speed measure of X, as well as its extended Dirichlet space; this is done in fact for an absorbing diffusion X^0, on a regular open interval with no killing inside. Possible symmetric extensions of X^0 are discussed. Finally, symmetric extensions of time changed transient reflecting Brownian motion on a special domain in $\mathbb{R}^d$ can be constructed by means of the Poisson point processes of excursions due to K. Itô.

N. Ikeda and S. Taniguchi recall the Itô–Nisio general representation of Brownian motion as a series expansion along any orthonormal basis of the Cameron–Martin space. They then go on discussing about P. Lévy's stochastic area for planar Brownian motion, and more generally Brownian quadratic functionals. They bring in a novel player in this domain by showing how Eulerian polynomials intervene in their computations.

H. Kunita reviews, assuming very little from the reader, the development of Itô's works from their very beginning:

- first, a Lévy process (X_t) may be realized from a Brownian motion and the Poisson random measure on $\mathbb{R}_+ \times (\mathbb{R} \setminus \{0\})$, which counts the jumps of X, as time evolves;
- second, Itô shows that one may represent a general jump diffusion driven by the tangential homogeneous Lévy processes, canonically attached to this jump diffusion;
- third, Itô's construction of stochastic integrals with respect to Brownian motions, and its subsequent generalizations due to Kunita–Watanabe are reviewed. Itô's formula and SDEs are also recalled;
- fourth, a general discussion of SDEs with respect to Brownian motions and Poisson random measures, as well as an associated Itô's formula is developed;
- fifth, in the third and last section of this paper, Itô's calculus is shown to apply remarkably well to Mathematical Finance (: the notions of assets, options and arbitrage are recalled), as the author derives the Black–Scholes partial differential equation as well as the study of equivalent martingale measures first for diffusions then for jump-diffusions, which are the underlying price processes.

S. Watanabe reviews Itô's theory of excursion point processes and shows how it applies to the study of one-dimensional diffusion processes on half-intervals which satisfy Feller's boundary conditions, and more generally, the study of multidimensional diffusion processes which satisfy Wentzell's boundary conditions.

J. Bertoin shows how Itô's excursion theory and the Lévy–Itô decomposition of subordinators are fundamental tools in his study of the genealogical structure of alleles for a Bienaymé–Galton–Watson branching process with neutral mutations. In particular, these tools enable J. Bertoin to establish a convergence in distribution to a certain continuous state branching process.

P. Biane highlights and explains the close connection that exists between, on one hand, the Itô formula for stochastic integrals, and, on the other hand, the Heisenberg uncertainty principle. He then illustrates this connection by describing the non commutative extension of Itô's calculus due to Hudson and Parthasarathy.

J.F. Le Gall shows precisely how the Itô excursion measure may be interpreted as the asymptotic distribution of the rescaled contour of a large Galton–Watson tree, thus obtaining a simple derivation of Aldous' theorem that the rescaled contour function, conditioned to have a fixed large progeny, converges to a normalized Brownian excursion. The author also obtains an analogous result when the tree is conditioned to have a fixed large height.

W. Werner explains how his famous works – together with G. Lawler and O. Schramm – which bear in particular upon the conformal invariance of planar Brownian motion have deep links with Itô's stochastic calculus on one hand, and Itô's excursion theory on the other hand: indeed, SLE processes may be described in terms of complex valued Bessel like processes, via Loewner's equation with driving real valued Brownian motion, and Werner also constructs Poisson point processes of SLE-type loops, which have a strong likeliness with Itô's Poisson point processes of, say, Brownian excursions.

To summarize, these eight papers provide at the same time a historical perspective as well as ongoing developments about both Itô's Stochastic Calculus and Itô's excursion theory.

These papers cover a lot of high level research material in Probability Theory; they are relatively short (between 20 and 30 pages), and, without relenting on rigor, bring the reader to the core of the topic. We believe these papers will remain as reference papers.

We very much appreciate Elsevier's decision of making this special issue freely available since its publication.

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